Task 4

Year 12 Unit 3 Gravity Test

Total Marks___/41 **Student:** Gravity Ans Key

1) A satellite is orbiting the Earth with a radius R and a period T. Determine by what factor (ratio) the period of the satellite will change if it is placed in orbit around the Moon and its orbital radius is ½ what it was originally. [6 marks]

$$T_E = \frac{4\pi^2 r^3}{GM_E}$$

$$\frac{M_M}{M_E} = \frac{7.35 \times 10^{22}}{5.97 \times 10^{24}} = 0.0123116$$

$$\frac{T_M}{T_E} = \frac{\sqrt{\frac{4\pi^2 (\frac{1}{2} r)^3}{G(0.0123116\, M_E)}}}{\sqrt{\frac{4\pi^2 r^3}{G M_E}}} = \frac{\sqrt{\frac{(\frac{1}{2})^3}{(0.0123116}}}{1}$$

$$\frac{T_M}{T_E} = \sqrt{\frac{0.125}{0.0123116}} = \sqrt{10.153} = 3.186$$

The period of the orbit around the moon will be 3.19 times longer. It is longer due to the lower mass. If the mass were the same as earth the period would decrease (be shorter).

Note: $T_E : T_M$ OR $T_M : T_E$

$0.31 : 1$ $3.186 : 1$

2) A 24.5 tonne rocket full of rocket fuel is to be launched into space.

a) What is the minimum possible kinetic energy needed to enable this rocket to achieve escape velocity? Note: Show how you derived the equation for escape velocity prior to doing your calculations to ensure full marks. [4 marks]

$$PE_i + KE_i = \cancel{KE_f} + \cancel{PE_f}$$
$$\qquad\qquad\qquad 0 \qquad 0$$

$$\therefore PE_i = -KE_i$$

$$-\frac{GMm}{r_E} = -\frac{1}{2}mv^2$$

$$\sqrt{\frac{2GM_E}{r_E}} = V = \sqrt{\frac{2(6.67\times10^{-11})(5.97\times10^{24})}{(6.37\times10^{6})}} = 11181\ m/s$$

$$V_{escape} = 1.12\times10^{4}\ m/s$$

$$\therefore KE = \frac{1}{2}mv^2 = \frac{1}{2}(24500)(11181)^2 = 1.53\times10^{12}\ J$$

b) If the scientists instead want to launch this rocket so that it delivers a satellite into a stable orbit at an altitude of 1250 km above the Earth's surface. How fast would the rocket need to be moving upwards from the Earth's surface initially? [4 marks]

$$PE_i + KE_i = \cancel{KE_f} + PE_f$$
$$\qquad\qquad\qquad 0$$

$$-\frac{GM_Em}{r_E} + \frac{1}{2}mv^2 = \frac{-GMm}{(1250000 + R_E)}$$

$$-\frac{(6.67\times10^{-11})(5.97\times10^{24})}{(6.38\times10^{6})} + \frac{1}{2}v^2 = -\frac{(6.67\times10^{-11})(5.97\times10^{24})}{(7.63\times10^{6})}$$

6.38   7.63

$$-6.2511616\times10^{7} + \frac{1}{2}v^2 = -5.2257086\times10^{7}$$

-6.2413636×10^{7} -5.2188597×10^{7}

$$\frac{1}{2}v^2 = 1.02546\times10^{7}$$

$$V = \sqrt{20509200}$$

20450076

$$v = 4.53\times10^{3}\ m/s$$

4.52×10^{3}

4.53×10^{3} m/s upward velocity is needed to reach this orbital radius.

c) If you increased the orbital radius of the satellite from 1250 km above the surface to 1820 km how much would the rockets:

i. Orbital velocity change (Note: provide the change in velocity) [4 marks]

$$\frac{\cancel{m}v^2}{\cancel{r}} = \frac{GM\cancel{m}}{r^{\cancel{2}}}$$

$$v_i = \sqrt{\frac{GM}{r_i}} \quad \text{vs} \quad v_f = \sqrt{\frac{GM}{r_f}}$$

$$v_i = \sqrt{\frac{(6.67\times10^{-11})(5.97\times10^{24})}{(6.38\times10^6 + 1.250\times10^6)}} \qquad v_f = \sqrt{\frac{(6.67\times10^{-11})(5.97\times10^{24})}{(6.38\times10^6 + 1.82\times10^6)}}$$

$$v_i = 7.224\times10^3 \text{ m/s} \qquad v_f = 6.968\times10^3 \text{ m/s}$$

$$\Delta v = v_f - v_i = 6.968\times10^3 - 7.224\times10^3$$

$\Delta v = -255$ m/s slows down by 255 m/s

ii. Period change: (Note: provide the change in period time) [4 marks]

$$T_i = \sqrt{\frac{4\pi^2 r_i^3}{GM}} \quad \text{vs} \quad T_f = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

$$T_i = \sqrt{\frac{4\pi^2(6.38\times10^6 + 1.25\times10^6)^3}{(6.67\times10^{-11})(5.97\times10^{24})}} \quad \text{vs.} \quad T_f = \sqrt{\frac{4\pi^2(6.38\times10^6 + 1.82\times10^6)^3}{(6.67\times10^{-11})(5.97\times10^{24})}}$$

$$T_i = 6636 \text{ s} \qquad T_f = 7393.5$$

$$\Delta T = T_f - T_i = 7393 - 6636$$

$\Delta T = 758$ s increase

12.6 min increase

3) What is your apparent weight if you are on the surface of the moon? Show your calculations.
[2 marks]

$$\cancel{m}\, g = \frac{GM \cancel{m}}{r^2}$$

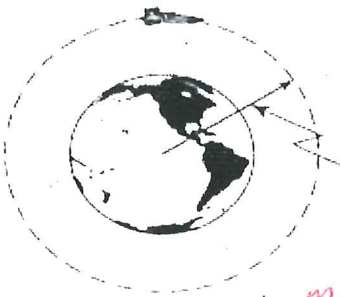
$$g = \frac{6.67\times10^{-11}(7.35\times10^{22})}{(1.74\times10^{6})^2}$$

$$g = 1.619\ m/s^2 \text{ or } \frac{N}{kg}$$

$$F_g = mg$$
$$= m(1.619)$$
$$= ?$$

4) A 2.45 x 10^4 kg space craft is orbiting the Earth at a radius of 8.95 x 10^6 m.

What is the centripetal acceleration of this space craft?
[**4 marks**]



R = 8.95 x 10^6 m

$$a_c = \frac{V_c^2}{r}$$

$$\frac{\cancel{m} V_c^2}{\cancel{r}} = \frac{GM\cancel{m}}{r^{\cancel{2}}}$$

$$V_c = \sqrt{\frac{GM}{r}}$$

$$V_c = \sqrt{\frac{(6.67\times10^{-11})(5.97\times10^{24})}{8.95\times10^{6}}}$$

$$V_c = 6670\ m/s$$

$$a_c = \frac{(6670)^2}{8.95\times10^6} = 4.97\ m/s^2$$

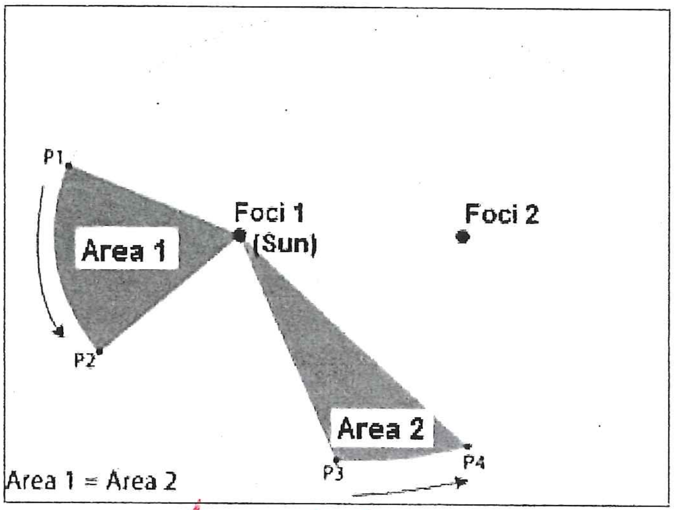
$$ma_c = mg$$

or

$$g = \frac{GM}{r^2}$$ at that radius

5) The area sectors labelled Area 1 and 2 are equal areas. Using Kepler's Laws to make the following comparisons:

a) Compare the velocity of the object as it moves from P1 to P2 versus when it moves from P3 to P4. Justify your answer. [**2 marks**]



The velocity is much slower from P3 to P4 since less distance is covered in the same time. Law of areas states that same area = same travel time

2

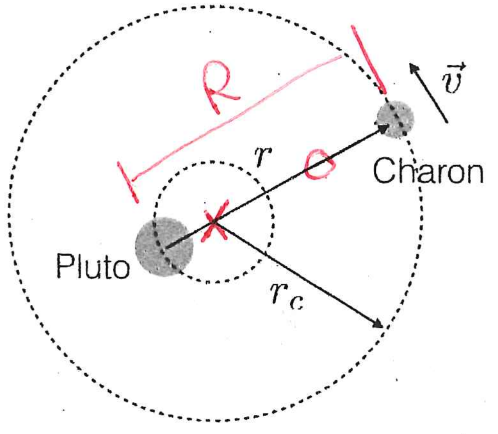
~~$T_{1\to2} = \sqrt{\frac{4\pi^2 r^3}{GM}}$ $T_{3\to4} \approx \sqrt{\frac{4\pi^2 r^3}{GM}}$ as radius increases T increases at $\sim \sqrt{r^3}$ or $r^{3/2}$~~

b) Compare the time taken for the object to move from P1 to P2 versus the time to move from P3 to P4. State which of Kepler's Laws you applied to answer this question. [**2 marks**]

2 Same period due to the law of areas.

Do not use

6) Refer to the diagram to the right then answer the following questions about Charon and Pluto. Note: Charon is a moon of Pluto.



a) Mark an **X** on the diagram where you think the centre of mass is for this system. [1 mark]

b) Mark a **O** on the diagram to indicate where you think you could achieve weightlessness in this system. [1 mark]

c) Using a ruler explain how you could estimate the mass of Charon relative to the mass of Pluto assuming this diagram is to scale. [2 marks]

Measure the distance between the centre of mass of the planet to the centre of mass of the system to get r for each planet. You can think of the centre of mass as a balance point ∴ $m_p(R - r_c) = m_c r_c$

$\frac{m_p}{m_c} = \frac{r_c}{(R - r_c)}$ 2 $= \frac{2.7}{0.8} m_c = 3.375 m_c = m_p$

or $m_c = \frac{1}{3.375} m_p = 0.3 m_p$

7) Physicists know that gravitational fields are not real. They now explain gravity by particle theory. Explain the following:

a) Why would physicists continue to use this theory when they know it is incorrect? [1 marks]

1 It is very useful mathematically to calculate values.

2 Explains how the g value changes with distance in a very simple way

b) What is meant by field strength? What are the units of a gravitational field? [2 marks]

1 It proportional to the intensity of the field at any given point. 2 $\frac{N}{Kg}$

OR it is the acceleration due to gravity $\frac{m}{s^2}$

c) Compare and contrast gravitational force and the gravitational field strength. State how they are similar and how they are different. [2 marks]

1 $F_g = m\left(\frac{GM}{r^2}\right)$ | $g = \frac{GM}{r^2}$

Both obey the inverse square law

1 The gravitational force depends on the mass that is moving in the field | independent of mass moving in the field.